Image resampling and constraint formulation for multi-frame super-resolution restoration

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Introduction

Consider the problem of multi-frame super-resolution image restoration:

- Estimate super-resolved images from multiple images of the scene
- Relative scene/camera motion provides essential constraints for restoration

Objectives:

- Generalize observation model
- Ideally no changes to restoration framework should be necessary
- Easy to incorporate spatially varying degradations
- Accommodate arbitrary motion fields

Overview

- Multi-frame super-resolution introduction
- Image resampling theory
- Multi-frame super-resolution observation model
- Show relationship between the two!
- Example observation model including lens PSF and pixel integration
- Example projected images and constraints
- Extensions

Multi-Frame Super-Resolution Restoration

- Given: noisy, under-sampled low-resolution image sequence
- Estimate: Super-resolved images (bandwidth extrapolation)
- Use information from multiple observed images in estimate
- Sub-pixel registration of multiple images[a] provides restoration constraints
- Model observation process (lens / sensor / noise)
- Include *a-priori* knowledge

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[a] Think IMAGE WARPING or RESAMPLING

Image Resampling

- Objective: Sampling of discrete image under coordinate transformation
- Discrete input image (texture): $f(\mathbf{u})$ with $\mathbf{u} = [u\ v]^T \in \mathbb{Z}^2$
- Discrete output image (warped): $g(\mathbf{x})$ with $\mathbf{x} = [x\ y]^T \in \mathbb{Z}^2$
- Forward mapping: $H : \mathbf{u} \longmapsto \mathbf{x}$
- Simplistic approach: $\forall \mathbf{x} \in \mathbb{Z}^2, \quad g(\mathbf{x}) = f(H^{-1}(\mathbf{x}))$
- Problems:
  1. $H^{-1}(\mathbf{x})$ need not fall on sample points (interpolation required)
  2. $H^{-1}(\mathbf{x})$ may undersample $f(\mathbf{u})$ resulting in aliasing

     (This occurs when the the mapping results in minification)

Theoretical Image Resampling Pipeline (Heckbert)

1. Continuous reconstruction (interpolation) of input image (texture):

$$\tilde{f}(\mathbf{u}) = f(\mathbf{u}) \circledast r(\mathbf{u}) = \sum_{\mathbf{k} \in \mathbb{Z}^2} f(\mathbf{k}) \cdot r(\mathbf{u} - \mathbf{k})$$

2. Warp the continuous reconstruction:

$$\tilde{g}(\mathbf{x}) = \tilde{f}\left(H^{-1}(\mathbf{x})\right)$$

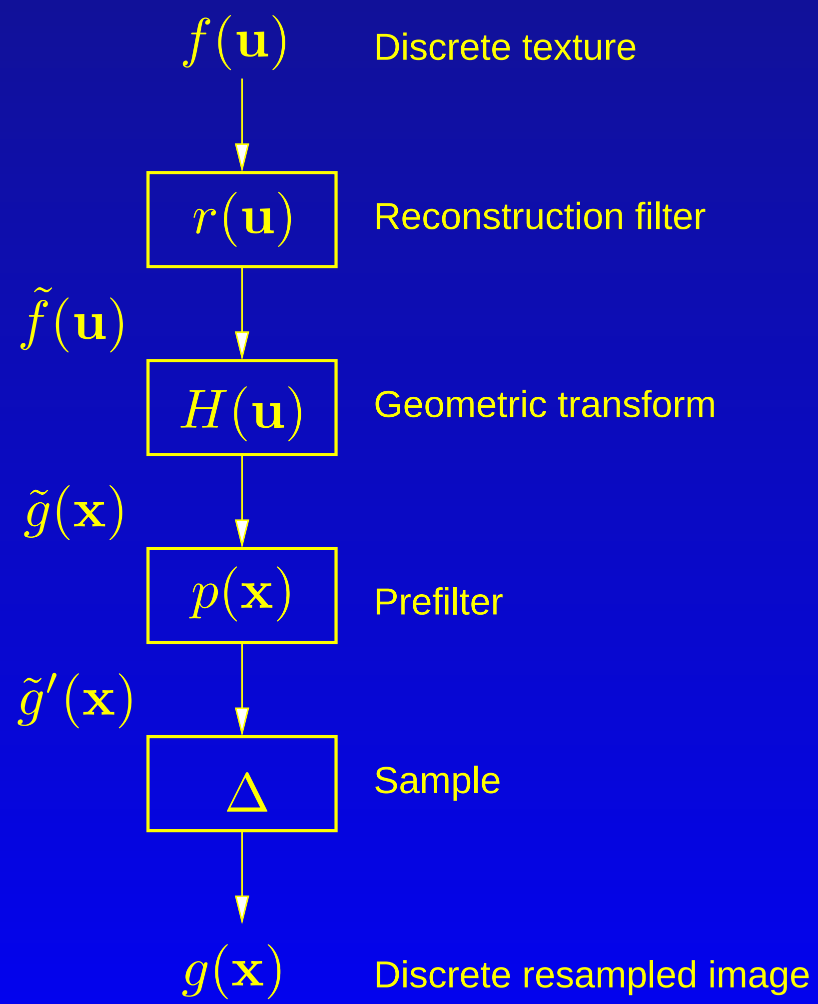
3. Pre-filter warped image to prevent aliasing in sampling step:

$$\tilde{g}'(\mathbf{x}) = \tilde{g}(\mathbf{x}) \circledast p(\mathbf{x}) = \iint \tilde{g}(\boldsymbol{\alpha}) \cdot p(\mathbf{x} - \boldsymbol{\alpha}) \, \mathrm{d}\boldsymbol{\alpha}$$

4. Sample to produce discrete output image:

$$g(\mathbf{x}) = \tilde{g}'(\mathbf{x}) \quad \text{for} \quad \mathbf{x} \in \mathbb{Z}^2$$

Heckbert's Resampling Pipeline



Realized Image Resampling Pipeline (Heckbert)

- Never reconstruct continuous images:

$$
\begin{aligned}
g(\mathbf{x}) &= \tilde{g}'(\mathbf{x}) \quad \text{for} \quad \mathbf{x} \in \mathbb{Z}^2 \\
&= \iint \tilde{f}\left(H^{-1}(\boldsymbol{\alpha})\right) \cdot p(\mathbf{x} - \boldsymbol{\alpha})\, \mathrm{d}\boldsymbol{\alpha} \\
&= \iint p(\mathbf{x} - \boldsymbol{\alpha}) \sum_{\mathbf{k} \in \mathbb{Z}^2} f(\mathbf{k}) \cdot r\left(H^{-1}(\boldsymbol{\alpha}) - \mathbf{k}\right) \mathrm{d}\boldsymbol{\alpha} \\
&= \sum_{\mathbf{k} \in \mathbb{Z}^2} f(\mathbf{k})\, \rho(\mathbf{x}, \mathbf{k})
\end{aligned}
$$

where

$$\rho(\mathbf{x}, \mathbf{k}) = \iint p(\mathbf{x} - \boldsymbol{\alpha}) \cdot r\left(H^{-1}(\boldsymbol{\alpha}) - \mathbf{k}\right) \mathrm{d}\boldsymbol{\alpha}$$

is a <u>spatially varying</u> resampling filter.

Realized Image Resampling Pipeline (Heckbert)

- The resampling filter

$$\rho(\mathbf{x},\mathbf{k}) = \iint p(\mathbf{x}-\boldsymbol{\alpha}) \cdot r\left(H^{-1}(\boldsymbol{\alpha}) - \mathbf{k}\right) \mathrm{d}\boldsymbol{\alpha}$$

  is described in terms of the warped reconstruction filter r and integration in $\mathbf{x}$-space.

- With a change of variables $\boldsymbol{\alpha} = H(\mathbf{u})$ and integrating in $\mathbf{u}$-space the resampling filter can be expressed in terms of the warped pre-filter p

$$\rho(\mathbf{x},\mathbf{k}) = \iint p(\mathbf{x} - H(\mathbf{u})) \cdot r\left(\mathbf{u} - \mathbf{k}\right) \left|\frac{\partial H}{\partial \mathbf{u}}\right| \mathrm{d}\mathbf{u}$$

- $|\partial H / \partial \mathbf{u}|$ is the determinant of the Jacobian

Super-Resolution Observation Model

Generalize super-resolution observation model of Schultz & Stevenson:

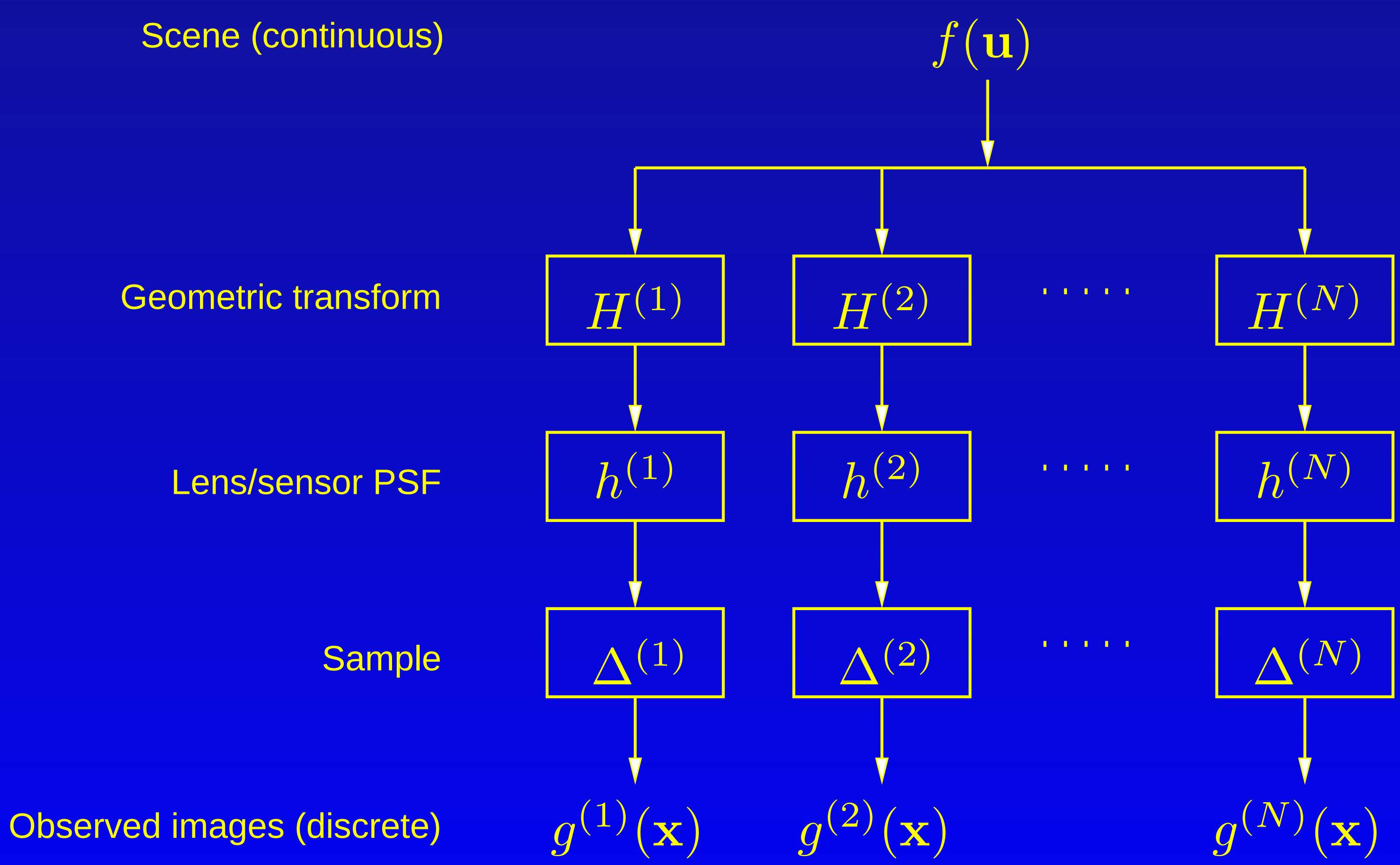
- Optics
  - Diffraction limited PSF, defocus, aberrations, etc...
- Sensor
  - Spatial reponse, temporal integration
- Must be able to accommodate spatially varying degradations
- Need technique for general motion maps!

Multi-Frame Observation Model

- N observations $g^{(i)}(\mathbf{x}),\ i \in \{1, 2, \ldots, N\}$ of underlying scene $f(\mathbf{u})$
- Related via geometric transformations $H^{(i)}$ (scene/camera motion)
- Spatially varying PSFs $h^{(i)}$ (may vary across observations)
- LSV PSFs can include lens and sensor responses, defocus, motion blur etc.

$$g^{(i)}(\mathbf{x})\Big|_{\mathbf{x}\in\mathbb{Z}^2} = \iint h^{(i)}(\mathbf{x}, \boldsymbol{\alpha}) \cdot f\left(H^{(i)^{-1}}(\boldsymbol{\alpha})\right) \mathrm{d}\boldsymbol{\alpha}$$

Multi-Frame Observation Model



Super-Resolution Restoration Observation Model

- Relate observations to image to be restored (estimate of scene)
- Discretized approximation $f(\mathbf{k})$ of scene $f(\mathbf{u})$ using interpolation kernel h_r

$$f(\mathbf{u}) \approx \sum_{\mathbf{k}} f(\mathbf{k}) \cdot h_r(\mathbf{u} - \mathbf{k})$$

- Combining with earlier result,

$$\begin{aligned} g^{(i)}(\mathbf{x})\Big|_{\mathbf{x}\in\mathbb{Z}^2} &= \iint h^{(i)}(\mathbf{x}, \boldsymbol{\alpha}) \cdot f\left(H^{(i)^{-1}}(\boldsymbol{\alpha})\right) \mathrm{d}\boldsymbol{\alpha} \\ &= \iint h^{(i)}(\mathbf{x}, \boldsymbol{\alpha}) \sum_{\mathbf{k}} f(\mathbf{k}) \cdot h_r\left(H^{(i)^{-1}}(\boldsymbol{\alpha}) - \mathbf{k}\right) \mathrm{d}\boldsymbol{\alpha} \end{aligned}$$

- Identical in form to resampling expressions
- Can thus find spatially variant resampling filter relating $g^{(i)}(\mathbf{x})$ to $f(\mathbf{k})$

Comparison of Resampling and Restoration Models

Resampling		Restoration	
Discrete texture	$f(\mathbf{u})$	$f(\mathbf{u})$	Discrete scene estimate
Reconstruction filter	$r(\mathbf{u})$	$h_r(\mathbf{u})$	Interpolation kernel
Geometric transform	$H(\mathbf{u})$	$H^{(i)}(\mathbf{u})$	Scene/camera motion
Anti-alias prefilter	$p(\mathbf{x})$	$h^{(i)}(\mathbf{x}, \boldsymbol{\alpha})$	Observation SVPSF
Warped output image	$g(\mathbf{x})$	$g^{(i)}(\mathbf{x})$	Observed images

- Resampling filter $\rho(\mathbf{x}, \mathbf{k}) = \iint p(\mathbf{x} - H(\mathbf{u})) \cdot r(\mathbf{u} - \mathbf{k}) \left| \frac{\partial H}{\partial \mathbf{u}} \right| d\mathbf{u}$
- Observation filter $\rho^{(i)}(\mathbf{x}, \mathbf{k}) = \iint h^{(i)}(\mathbf{x}, H(\mathbf{u})) \cdot h_r(\mathbf{u} - \mathbf{k}) \left| \frac{\partial H^{(i)}}{\partial \mathbf{u}} \right| d\mathbf{u}$
- But how do we find the observation filter in practice?

Determining the Observation Filter

- Measure or model combined PSFs
  - Lens, sensor spatial integration, temporal integration, defocus, etc.
- Estimate or model inter-frame registration (geometric transforms)
  - Motion estimation from observed scenes, observation geometry, etc.
- Present an example:
  - PSF accounts for diffraction limited optical system and sensor spatial integration
  - Geometric transforms based on controlled imaging geometry
  - Demonstrate method for finding observation filter

Optical System Modeling

Assumptions:

- Diffraction limited
- Incoherent illumination
- Circular exit pupil

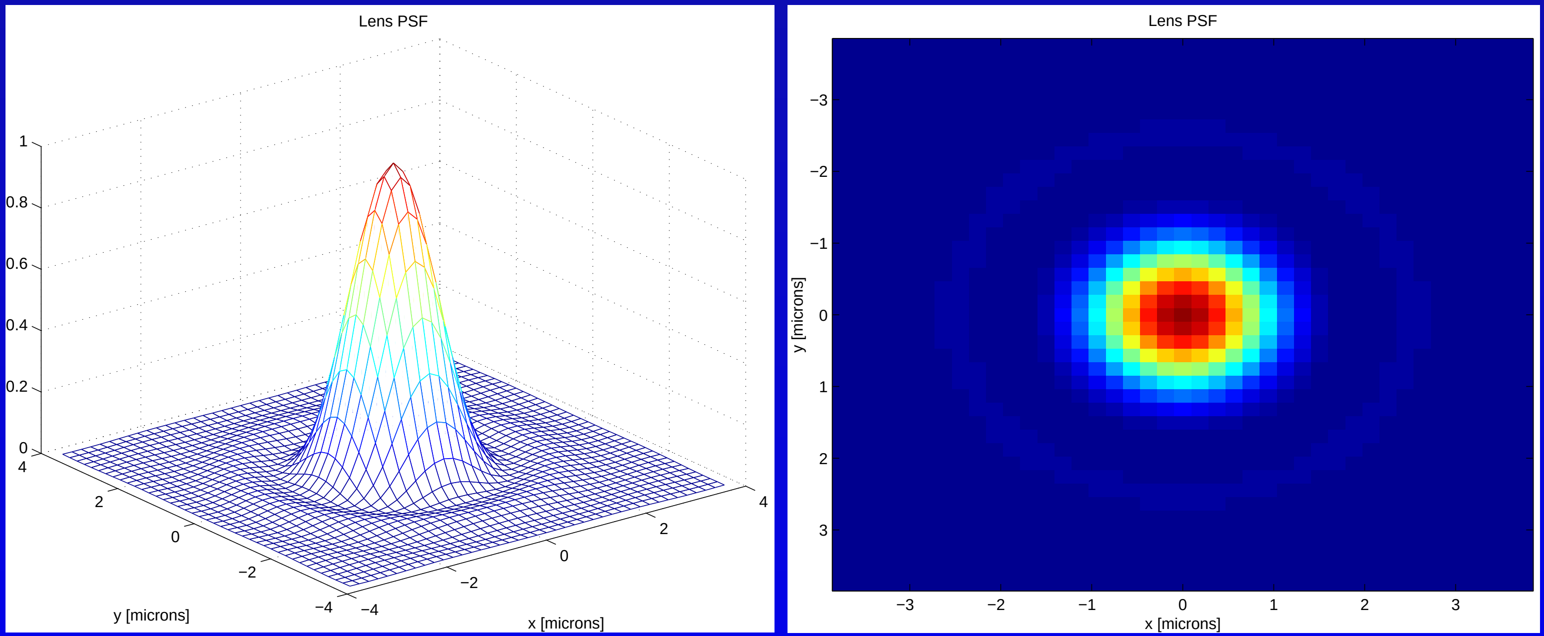
$\Rightarrow$ Radially symmetric point spread function

$$h(r') = \left[2 \frac{J_1(r')}{r'} \right]^2, \text{ with } r' = (\pi / \lambda N) r$$

$J_1(\cdot)$ Bessel function first kind, wavelength λ, f-number N, radial distance r.

Example – Optical System PSF

- λ=550nm (green), $N = 2.8$
- First zero of Airy Disk at $1.22\lambda N = 1.88\mu$m



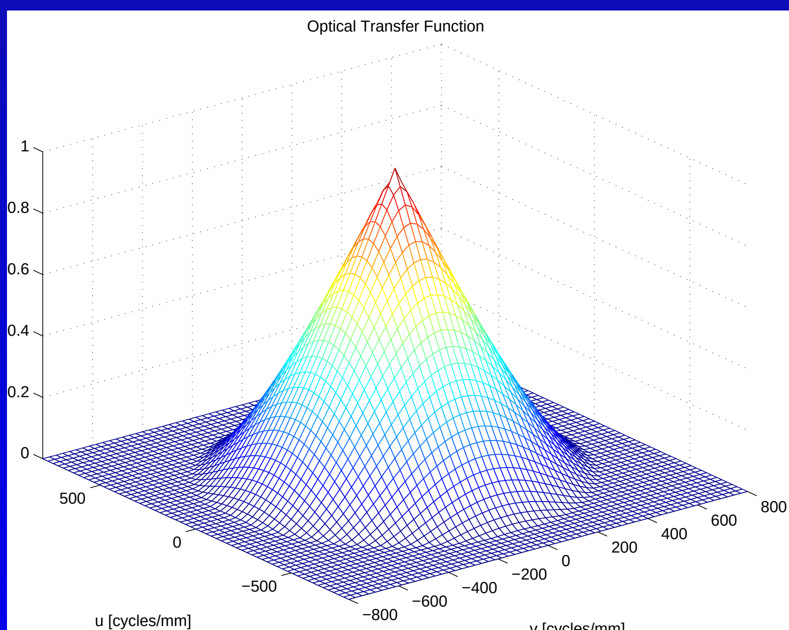
Optical Transfer Function

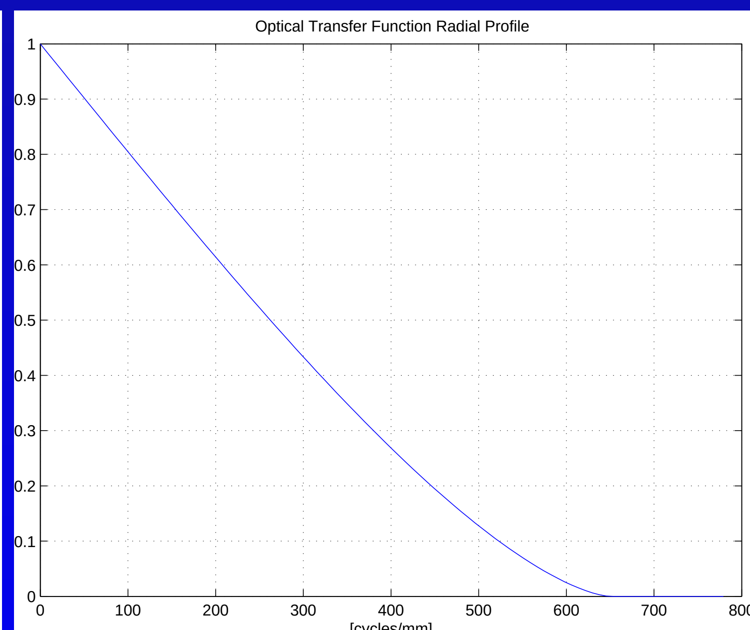
$$H(\rho') = \begin{cases} \frac{2}{\pi}\left[\cos^{-1}(\rho') - \rho'\sqrt{1-\rho'^2}\right], & \text{for } \rho' \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

$\rho' = \rho/\rho_c$	normalized radial spatial frequency
ρ	radial spatial frequency
$\rho_c = 1/\lambda N$	radial spatial frequency cut-off

Example – Optical Transfer Function

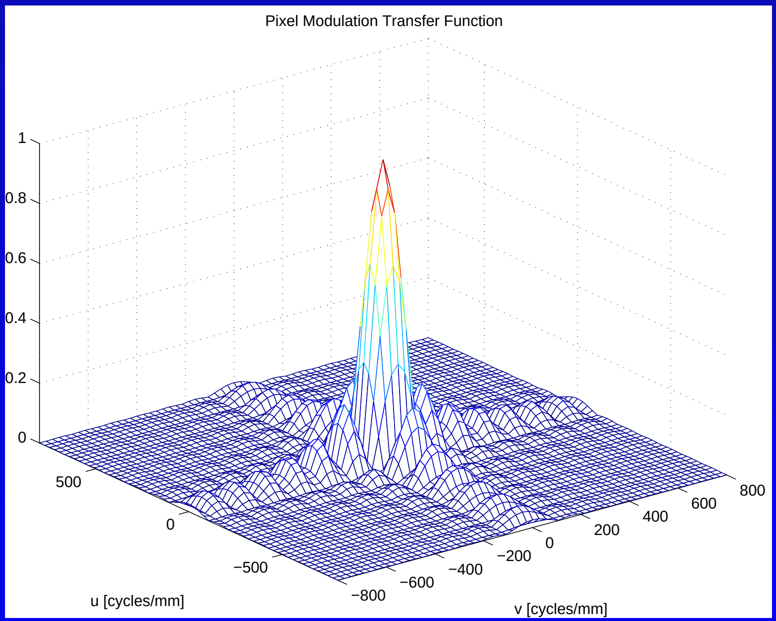
- Radial frequency cut-off $\rho_c = 1/\lambda N = 649.35$ lines/mm
- Nyquist sampling requires $> 2 \times \rho_c \approx 1300$ samples/mm
- Sample spacing $< 0.77\mu$m! ... (but pixel spatial integration is crude LPF)

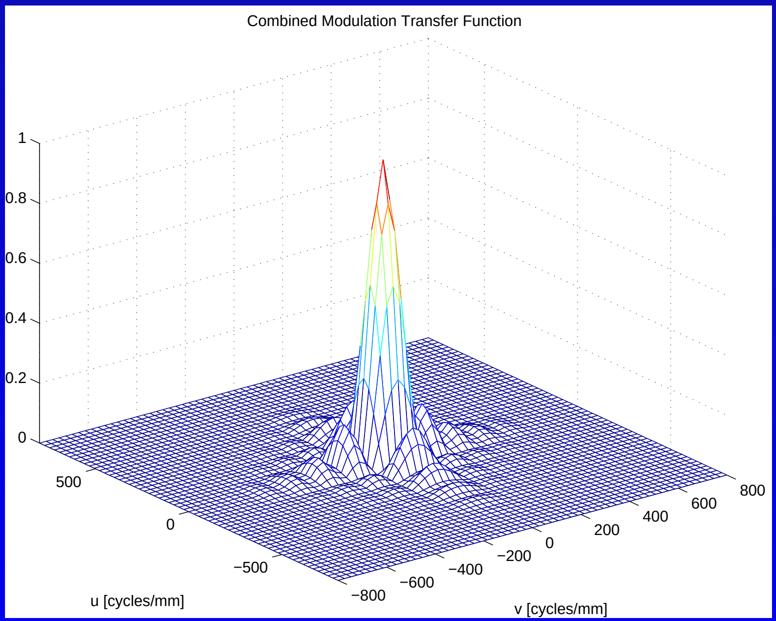




Combined Transfer Function

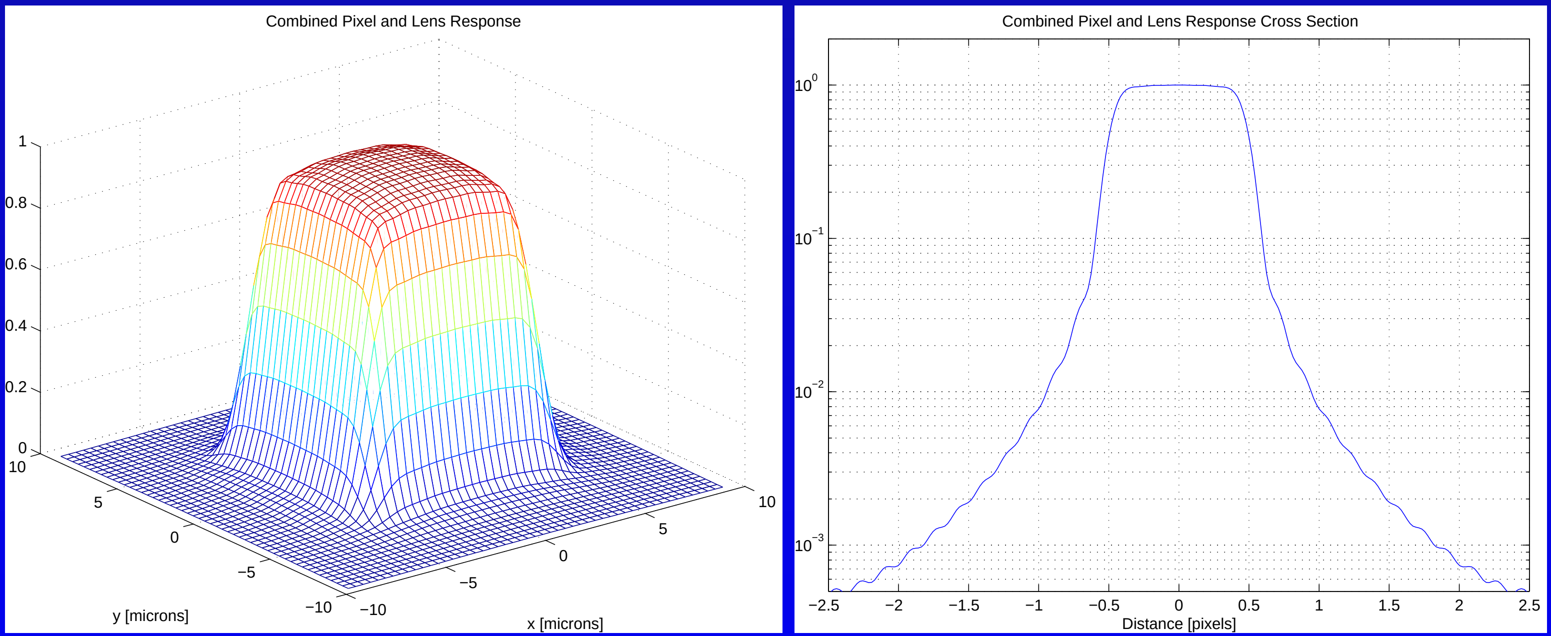
- LPF due to spatial integration over light-sensitive area of pixel
- Assume ideal pixel response (100% fill-factor, flat response, dimension T)
- Sinc from pixel integration has first zeros at sampling frequency ($f_s = 1/T$)
- Need zero response for $f > f_s/2$ for correct anti-aliasing





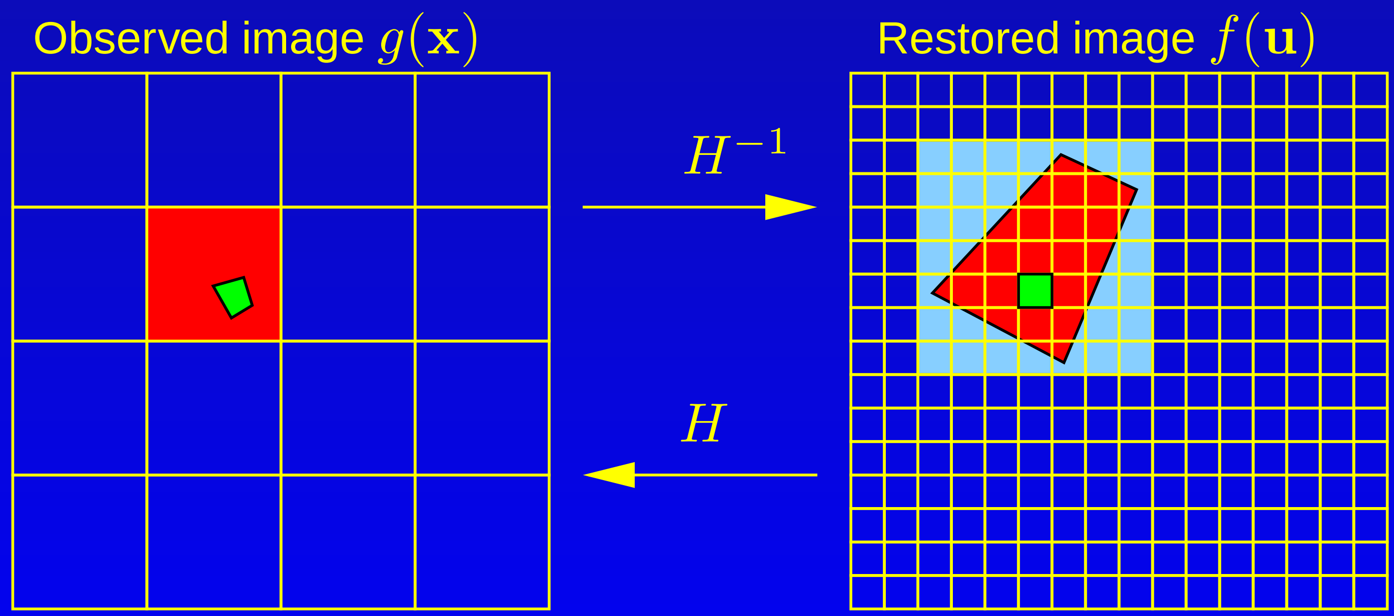
Combined Pixel Response Function

- $h_{combined} = h_{lens} * h_{pixel}$
- Significant "leakage" even with ideal lens/pixel responses
- Reality is much worse (optics and sensors)



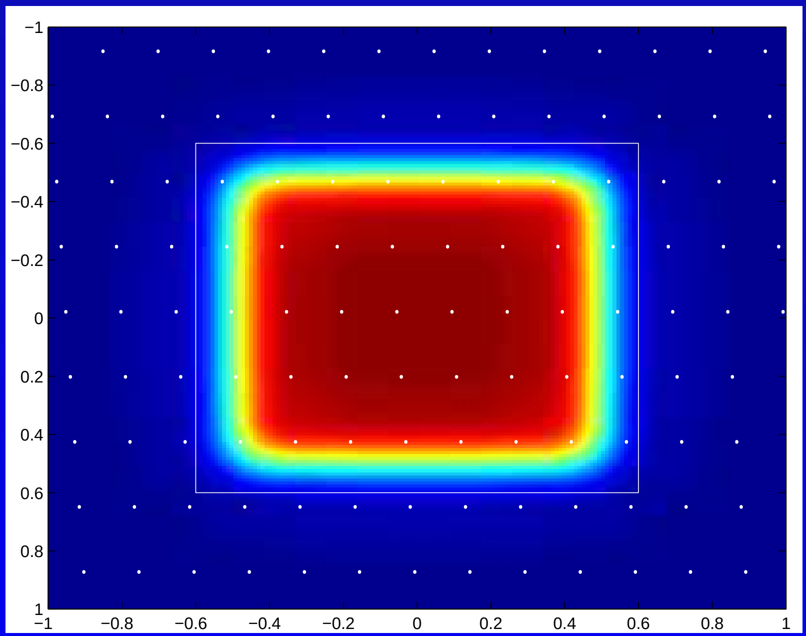
Determining the Warped Pixel Response

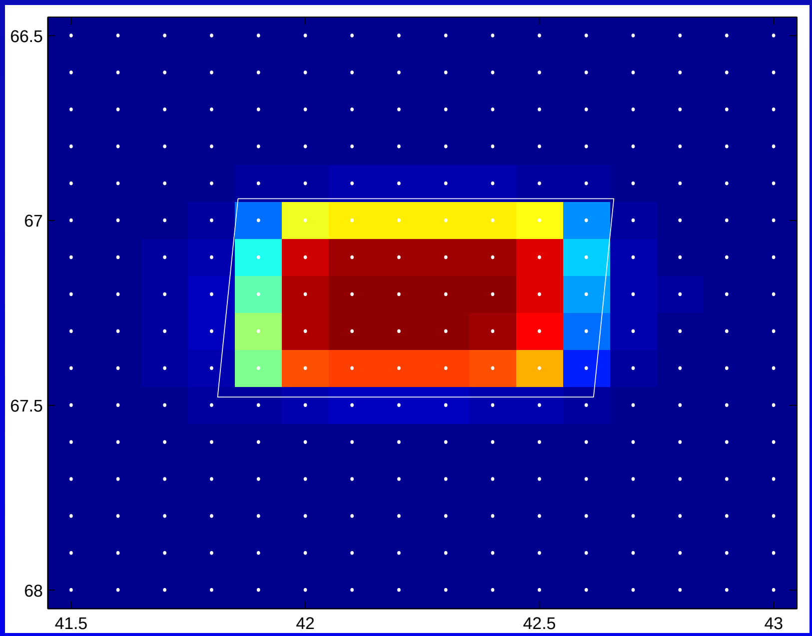
- Backproject PSF $h(\mathbf{x}, \boldsymbol{\alpha})$ from $g(\mathbf{x})$ to restored image using H^{-1} (red)
- Determine bounding region for image of $h(\mathbf{x}, \boldsymbol{\alpha})$ under backprojection (cyan)
- $\forall$ S-R pixels $\mathbf{u}$ in region, project via H and find $h(\mathbf{x}, H(\mathbf{u}))$ (green)
- Scale according to Jacobian and interpolation kernel h_r then integrate over $\mathbf{u}$



Example – Warped Pixel Response

- Projective transformation H
- PSF associated with observed pixel at location $(x, y) = (0, 0)$ (left)
- Regularly sampled warped PSF on restoration grid (u, v) (right)
- 10 samples/restoration pixel (right). Projection of samples shown (left)





Example – Projective Transformation

- Projective transformation H for geometric mapping

$$\begin{aligned} x &= \phi(u,v) &= \frac{h_{11}u+h_{12}v+h_{13}}{h_{31}u+h_{32}v+1} \\ y &= \psi(u,v) &= \frac{h_{21}u+h_{22}v+h_{23}}{h_{31}u+h_{32}v+1} \end{aligned}$$

- Linear when using homogeneous coordinates

$$\begin{bmatrix} xw \\ yw \\ w \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

- Jacobian

$$\mathrm{d}x\,\mathrm{d}y = \begin{vmatrix} \frac{\partial \phi}{\partial u} & \frac{\partial \phi}{\partial v} \\ \frac{\partial \psi}{\partial u} & \frac{\partial \psi}{\partial v} \end{vmatrix} \mathrm{d}u\,\mathrm{d}v$$

Discrete Observation Model

- Pixels in each observed image are related to the restored image

$$g^{(i)}(\mathbf{x})\Big|_{\mathbf{x}\in\mathbb{Z}^2} = \sum_{\mathbf{k}\in\mathbb{Z}^2} f(\mathbf{k})\, \rho^{(i)}(\mathbf{x},\mathbf{k})$$

- In the S-R restoration this constitutes one row of the observation matrix $\boldsymbol{A}$

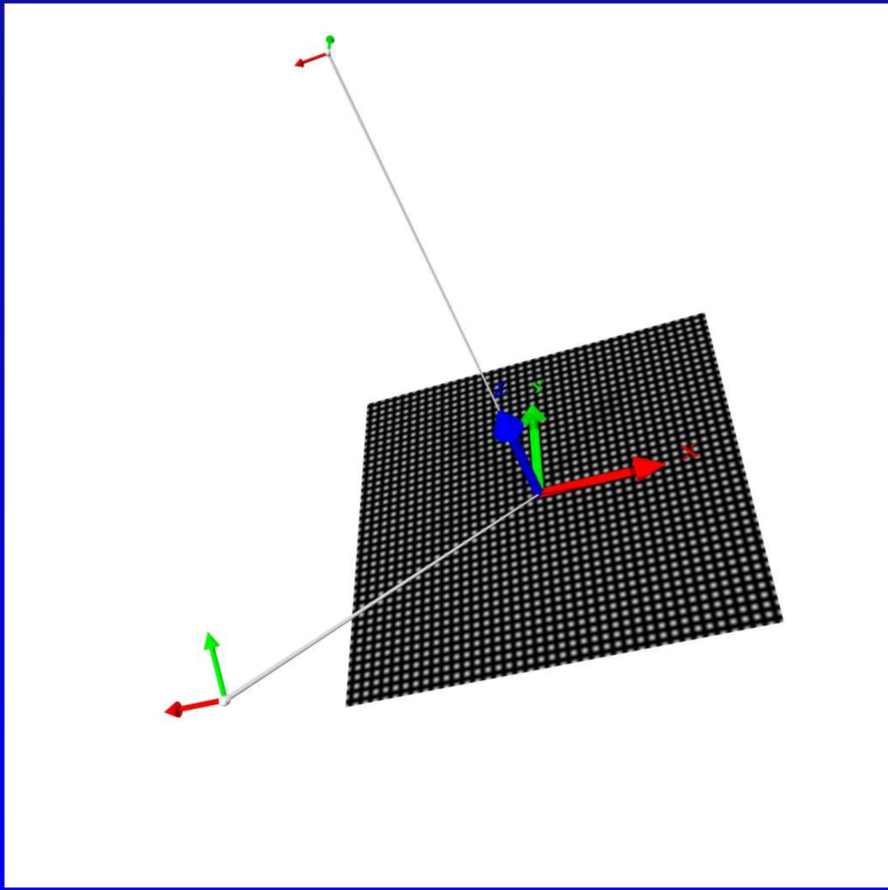
$$\left[\boldsymbol{g}^{(1)^T}\, \boldsymbol{g}^{(2)^T}\, \cdots\, \boldsymbol{g}^{(N)^T}\right]^T = \boldsymbol{A}\boldsymbol{f}$$

where the images are lexicographically ordered column vectors

- These equations are solved to determine the S-R estimate $\boldsymbol{f}$
- Typically regularized solution methods are used (ill-posed inverse problem)
- No changes to restoration algorithm are required!

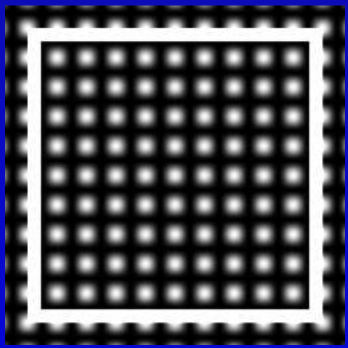
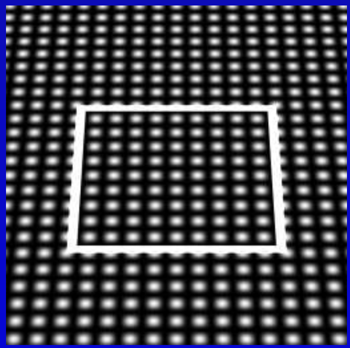
Example Demonstrating Observation Filter

- Create two simulated views with custom raytracer (with lens/sensor model)



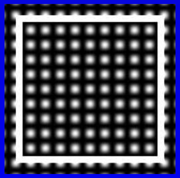
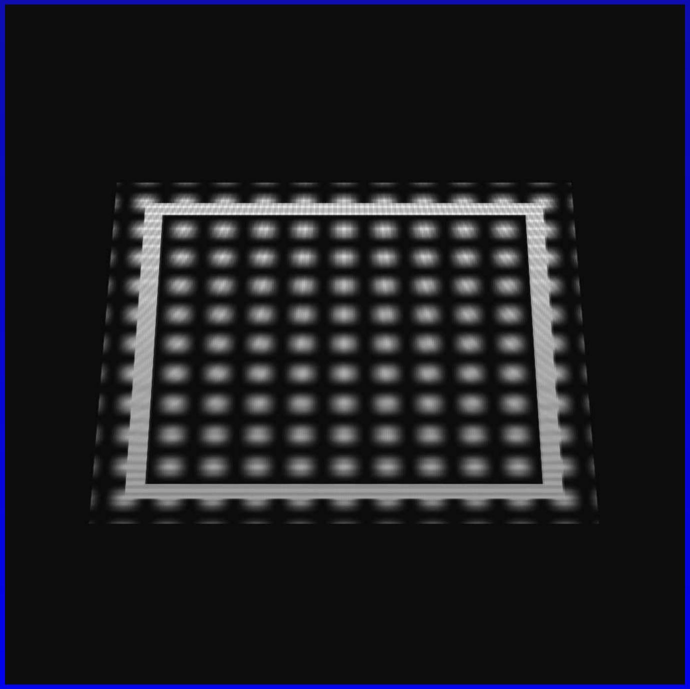
Example ctnd...

- Assume 2D “scene” in X-Y plane, lens/sensor PSF discussed earlier
- Camera matrices known, so compute homography induced by the plane
- Projective transformation H relates the image pair $\boldsymbol{g}^{(1)}$ and $\boldsymbol{g}^{(2)}$

$\boldsymbol{g}^{(1)}$   $\boldsymbol{g}^{(2)}$

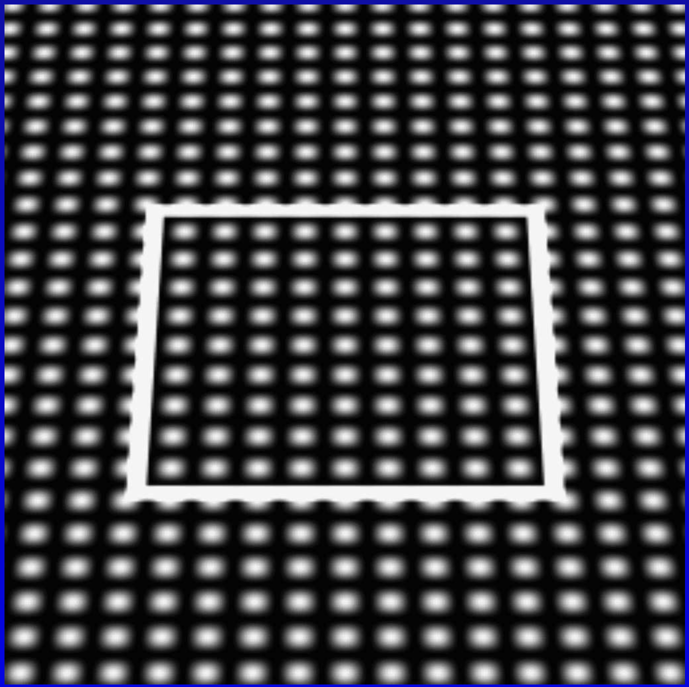
Example ctnd...

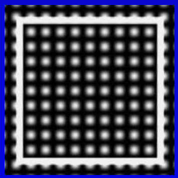
- Compute $\rho(\mathbf{x}, \mathbf{k})$ relating first (plan) view $\boldsymbol{g}^{(1)}$ to the restoration coincident with second (oblique) view $\boldsymbol{g}^{(2)}$ assuming 4:1 image expansion for restoration
- Compute backprojection $\boldsymbol{A}^T\boldsymbol{g}^{(1)}$ (should be oblique, expanded)

$\boldsymbol{g}^{(1)}$   $\boldsymbol{A}^T\boldsymbol{g}^{(1)}$

Example ctnd...

- Given estimate $\hat{f}$ of f compute projection $A\hat{f}$ (should look like $g^{(1)}$)

$\hat{f}$ 

 $A\hat{f}$

Features

- Applicable to general, spatially varying PSFs
- Spatially varying responses present no additional difficulties
- Measured PSF response data are easily incorporated
- Once $\rho^{(i)}(\mathbf{x}, \mathbf{k})$ are determined, restoration proceeds as usual
- $\boldsymbol{A}$-matrix projection/backprojection is completely characterized by $\rho^{(i)}(\mathbf{x}, \mathbf{k})$
- No change whatsoever to super-resolution restoration framework
- More realistic observation models $\Rightarrow$ better restoration

Extensions and Challenges

- Accelerate computing of $\rho^{(i)}(\mathbf{x}, \mathbf{k})$
  - Computing the projected PSFs is costly
  - Immediate saving: tighter bounding box on backprojection of PSF
  - Use scanline algorithms from CG literature
- Accuracy of observation model vs space complexity of storing $\rho^{(i)}(\mathbf{x}, \mathbf{k})$
- General image motion
  - Displacement vector field or locally modeled regions
  - Either way we have a local coordinate transformation (no problem!)
  - Occlusions?

Summary

- Close relationship between image resampling pipeline and the multi-frame image restoration problem
- Used ideas from resampling to develop method for incorporating very general observation models in the restoration framework
- Illustrated application to restoration with non-ideal lens/sensor response

The end...

"If it wasn't for the last minute, nothing would get done..."