

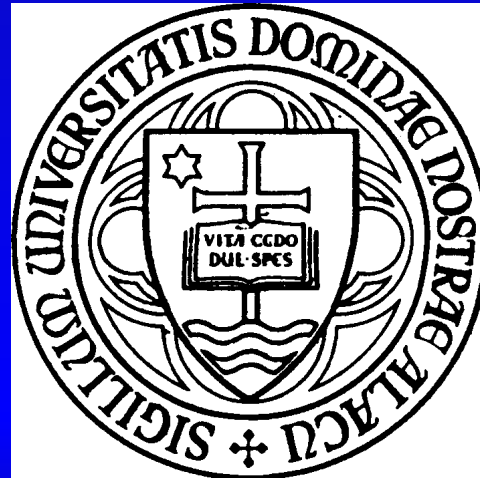
# Super-Resolution from Image Sequences

## A Review

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# Introduction

- Seminal work by Tsai and Huang 1984
- More information in a sequence than a single frame  
Also include *a-priori* info for BW extrapolation
- SR is an ill-posed inverse problem  
(regularized solution methods needed)
- Two main classes of SR algorithm
  1. Frequency domain
  2. Spatial domain

# Frequency Domain SR Methods

- Based on three principles:
  1. Shifting property of Fourier transform
  2. Alias relationship between DFT and CFT
  3. Scene is assumed bandlimited
- $\rightarrow$  system of equations relating aliased DFT coefficients of LR images to samples of the CFT of unknown scene
- Solving system  $\leftrightarrow$  De-aliasing

# The Details

Continuous scene:  $f(x, y)$

CFT:  $\mathcal{F}(u, v)$

Translations:  $f_r(x, y) = f(x + \Delta x_r, y + \Delta y_r)$

CFT:  $\mathcal{F}_r(u, v)$  with  $r = 1, 2, \dots, R$

Observation:  $y_r[m, n] = f(mT_x + \Delta x_r, nT_y + \Delta y_r)$

$$m = 0, 1, \dots, M - 1$$

$$n = 0, 1, \dots, N - 1$$

DFT:  $\mathcal{Y}_r[k, l]$

Aliasing:

$$\mathcal{Y}_r[k, l] = \frac{1}{T_x T_y} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \mathcal{F}_r \left( \frac{k}{MT_x} + pf_{s_x}, \frac{l}{NT_y} + qf_{s_y} \right)$$

Shifting:

$$\mathcal{F}_r(u, v) = e^{j2\pi(\Delta x_r u + \Delta y_r v)} \mathcal{F}(u, v)$$

If  $f(x, y)$  is band-limited above may be combined...

$$\mathbf{Y} = \Phi \mathbf{F}$$

$\mathbf{Y}$ : vector of observation image DFT's

$\Phi$ : matrix

$\mathbf{F}$ : vector of CFT coefficients (unknown)

Solve for  $\mathbf{F}$ , take inverse DFT for SR image

# Spatial Domain SR Methods

- Interpolation of non-uniformly spaced samples
- Iterated backprojection
- Stochastic methods
- Set theoretic methods
- Hybrid stochastic / set theoretic
- Other methods

# Observation Model

Images are lexicographically ordered.

LR images:  $\mathbf{y}_r, r \in \{1, 2, \dots, R\}$

SR image:  $\mathbf{z}$

Model:  $\mathbf{y}_r = \mathbf{H}_r \mathbf{z}$

General:  $\mathbf{Y} = \mathbf{H} \mathbf{z} + \mathbf{N}$

$$\mathbf{Y} = \begin{bmatrix} \mathbf{y}_1^T & \cdots & \mathbf{y}_R^T \end{bmatrix}^T$$
$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1^T & \cdots & \mathbf{H}_R^T \end{bmatrix}^T$$

Noise:  $\mathbf{N}$

# Interpolation of non-uniformly spaced samples

- Register LR frames yielding dense composite image of non-uniformly spaced samples
- SR image reconstructed from composite
- Too simplistic
- Limited de-aliasing. Poor incorporation of *a-priori* constraints. Limited degradation models. Separate merging and restoration suboptimal.

# Iterated backprojection

Simulate the LR images  $\hat{\mathbf{Y}}$  as  $\hat{\mathbf{Y}} = \mathbf{H}\hat{\mathbf{z}}$ .

Iteratively backproject error and correct the SR estimate:

$$\begin{aligned}\hat{\mathbf{z}}^{(j+1)} &= \hat{\mathbf{z}}^{(j)} + \mathbf{H}^{BP} \left( \mathbf{Y} - \hat{\mathbf{Y}}^{(j)} \right) \\ &= \hat{\mathbf{z}}^{(j)} + \mathbf{H}^{BP} \left( \mathbf{Y} - \mathbf{H}\hat{\mathbf{z}}^{(j)} \right).\end{aligned}$$

Problems:

- Non-uniqueness of solution
- Inclusion of *a-priori* constraints difficult

# Stochastic methods

- SR reconstruction as a statistical estimation problem
- Bayesian framework enables inclusion of *a-priori* info
- Stochastic observation equation  $\mathbf{Y} = \mathbf{H}\mathbf{z} + \mathbf{N}$
- Maximum A-Posteriori (MAP) estimate

$$\begin{aligned}\hat{\mathbf{z}}_{\text{MAP}} &= \arg \max_{\mathbf{z}} [\Pr \{\mathbf{z} | \mathbf{Y}\}] \\ &= \arg \max_{\mathbf{z}} [\log \Pr \{\mathbf{Y} | \mathbf{z}\} + \log \Pr \{\mathbf{z}\}].\end{aligned}$$

# Stochastic methods - MAP

$\log \Pr \{ \mathbf{Y} | \mathbf{z} \}$       Log-likelihood function

$\Pr \{ \mathbf{z} \}$               Prior density on  $\mathbf{z}$

- $\Pr \{ \mathbf{Y} | \mathbf{z} \} = f_{\mathbf{N}} (\mathbf{Y} - \mathbf{H}\mathbf{z})$  (noise PDF)
- $\Pr \{ \mathbf{z} \}$  is typically a MRF
- Gaussian noise and convex priors imply convex optimization

# Set theoretic methods

- Define constraint sets in space of SR image
- Solution is intersection of constraint sets
- Sets include data fidelity, positivity, bounded energy etc.
- Convex constraint sets allows use of the Projection Onto Convex Sets (POCS) algorithm
- Define constraint sets  $\mathcal{C}_\alpha$  and corresponding projection operators  $\mathcal{P}_\alpha$
- $\mathbf{z}^{(n+1)} = \mathcal{P}_1 \mathcal{P}_2 \mathcal{P}_3 \cdots \mathcal{P}_K \mathbf{z}^{(n)}$  (POCS)

# Hybrid stochastic / set theoretic

- Best of both worlds
- Stochastic: theoretical framework, uniqueness of solution, prior densities.
- Set theoretic: convenient *a-priori* constraints
- Maximize *a-posteriori* density / likelihood function subject to satisfying convex constraint sets
- Excellent incorporation of prior info

## Other methods

- Optimal and adaptive filtering
- Tikhonov-Arsenin regularization
- Few advantages not already provided by either Bayesian or POCS methods.

# Frequency vs Spatial Domain SR

	Frequency Domain	Spatial Domain
Observation model	Frequency domain	Spatial domain
Motion models	Global translation	Almost unlimited
Degradation model	Limited, LSI	LSI or LSV
Noise model	Limited, SI	Very Flexible
SR Mechanism	De-aliasing	De-aliasing <i>A-priori</i> info

# Frequency vs Spatial Domain SR

	Frequency Domain	Spatial Domain
Computation req.	Low	High
<i>A-priori</i> info	Limited	Almost unlimited
Regularization	Limited	Excellent
Extensibility	Poor	Excellent
Applicability	Limited	Wide
App. performance	Good	Good

# MAP vs. POCS SR

	Bayesian (MAP)	POCS
Applicable theory	Vast	Limited
A-priori info	Prior PDF Easy to incorporate No hard constraints	Convex Sets Easy to incorporate Powerful yet simple
SR solution	Unique MAP estimate	Non-unique $\cap$ of constraint sets

# MAP vs. POCS SR

	Bayesian (MAP)	POCS
Optimization	Iterative	Iterative
Convergence	Good	Possibly slow
Computation req.	High	High
Complications	Optimization under non-convex priors	Defn. of projection operators

# Directions for Future Research

1. Motion Estimation
2. Degradation Models
3. Restoration Algorithms

# Motion Estimation

- Holy Grail: arbitrary scenes. multiple independent motion, occlusions, transparency etc.
- Critically dependent on robust, model based, sub-pixel accuracy motion estimation
- Open research problem
- Motion estimated from the observed *undersampled* data
  - Reliability issues; Reliability measures ?

# Motion Estimation

- Constrained motion estimation for consistent motion maps – Regularized motion estimation
- Sparse maps: accurate motion estimates in areas of high spatial variance (locale of best SR enhancement)
- Independent model based motion predictors and estimators
- Simultaneous multi-frame motion estimation
- Simultaneous motion estimation and reconstruction

# Degradation Models

- Accurate observation model → improved reconstruction
- Color SR – model correlations, degradations
- Lossy Compression – color subsampling, quantization, blocking effects
- Magnetic Media – recording and playback degradations
- CCD arrays – model real devices  
sensor geometry, spatio-temporal integration, noise, readout effects

# Restoration Algorithms

- Hybrid MAP / POCS

MAP – Mathematical rigor and uniqueness of solution

POCS – Convenient *a-priori* constraints

- Simultaneous motion estimation and restoration

- Simultaneous multi-frame SR restoration