Block-Matching Sub-Pixel Motion Estimation from Noisy, Under-Sampled Frames – An Empirical Performance Evaluation

S. Borman, M. A. Robertson, R. L. Stevenson

Department of Electrical Engineering

University of Notre Dame

Wed. 27 January 1999

1. Introduction

- Why: Multi-frame super-resolution image reconstruction
- Multiple, noisy undersampled frames
  $\longrightarrow$ high-resolution frame
- Reliable sub-pixel motion necessary
- How do motion estimators perform under adverse conditions?
- Want: realistic best case performance

- Attack
  - Simulate a realistic imaging system
  - Use synthetic scene with known motion
  - Estimate motion, analyze errors

2. Imaging System Modeling

- Optical System
  - Diffraction limited, circular aperture
  - f/number 2.8
  - $\lambda = 670$nm
- CCD Focal Plane Array
  - Square pixels with dimension $X_s = 9\mu$m
  - Active region is entire pixel
  - Short aperture time
  - Additive Gaussian readout noise

- Sampling Considerations
  - Sampling:

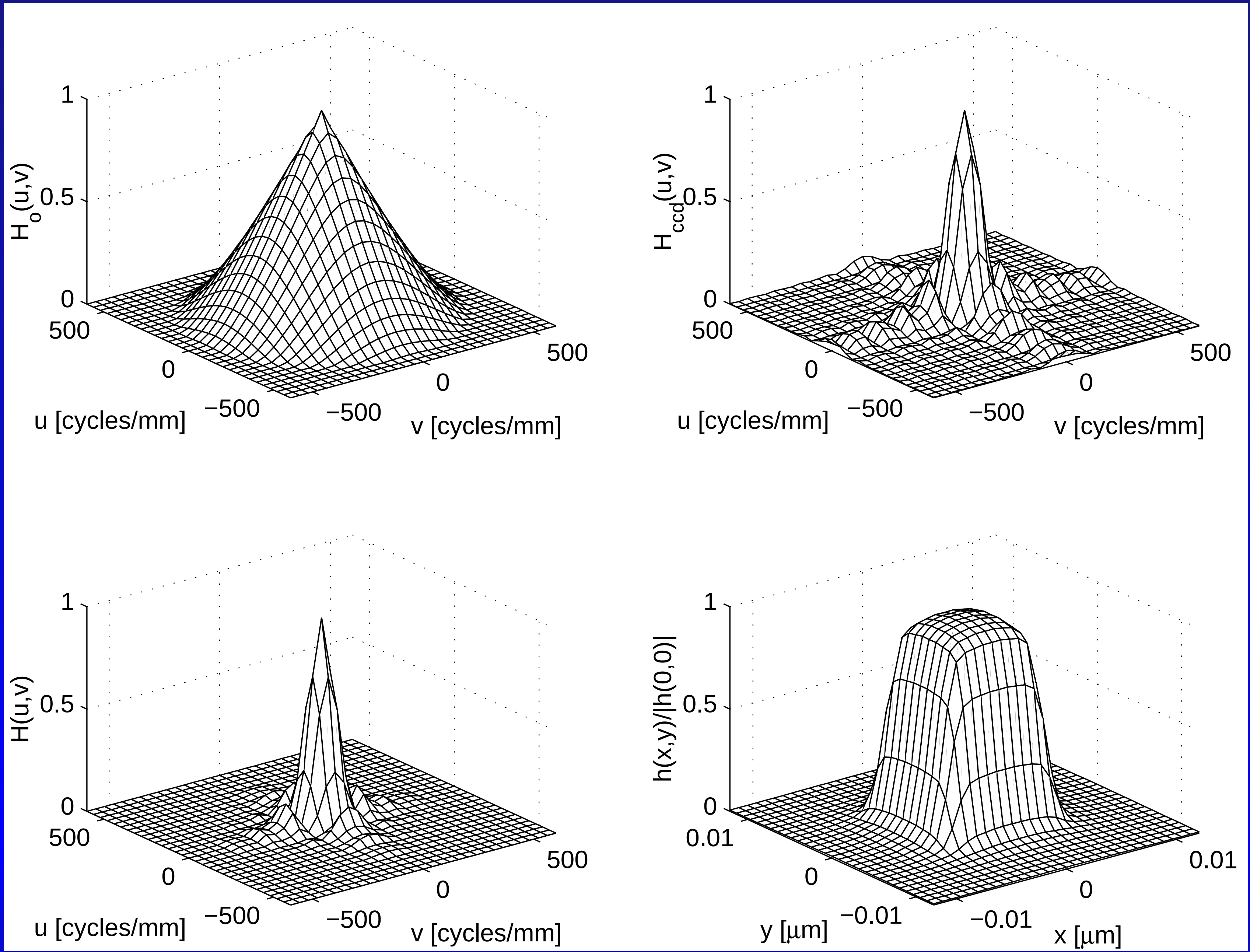
    CCD *spacing* determines sampling rate

    Not active region (aperture) of sensor
  - Sources of low-pass filtering:

    Optical System response (High f_c)

    CCD spatial integration (sinc, $f_0 = k\frac{1}{X_s}$)

$\Rightarrow$ Undersampled



1
0.5
0
$H_o(u,v)$
500
0
−500
u [cycles/mm]
−500
0
500
v [cycles/mm]
$H_{ccd}(u,v)$
$H(u,v)$
h(x,y)/|h(0,0)|
0.01
0
−0.01
y [µm]
x [µm]

3. Synthesized Scene Data

- Requirements

1. Need known scene motion
2. Seek “realistic best case” performance
    - Consider local translational motion model
    - 1-d “scene”
    - Choose realistic scene (typical in images)
    - Yields very reliable motion estimates

- Continuous-Space Scene Model
  - Use step edge $s(x)$, with simple texture $g(x)$

$$
\begin{aligned}
f(x, t_1) &= s(x) + g(x) \\
f(x, t_2) &= f(x - X_m, t_1) \\
&= s(x - X_m) + g(x - X_m)
\end{aligned}
$$

  - Translation parameter X_m – to be estimated

- Discrete-Space Approximation
  - Sampling of continous scene model

$$
\begin{aligned}
f[k,t_1] &= s(k\Delta) + g(k\Delta) \\
f[k,t_2] &= f[k-d,t_1] \\
&= s(k\Delta - X'_m) + g(k\Delta - X'_m)
\end{aligned}
$$

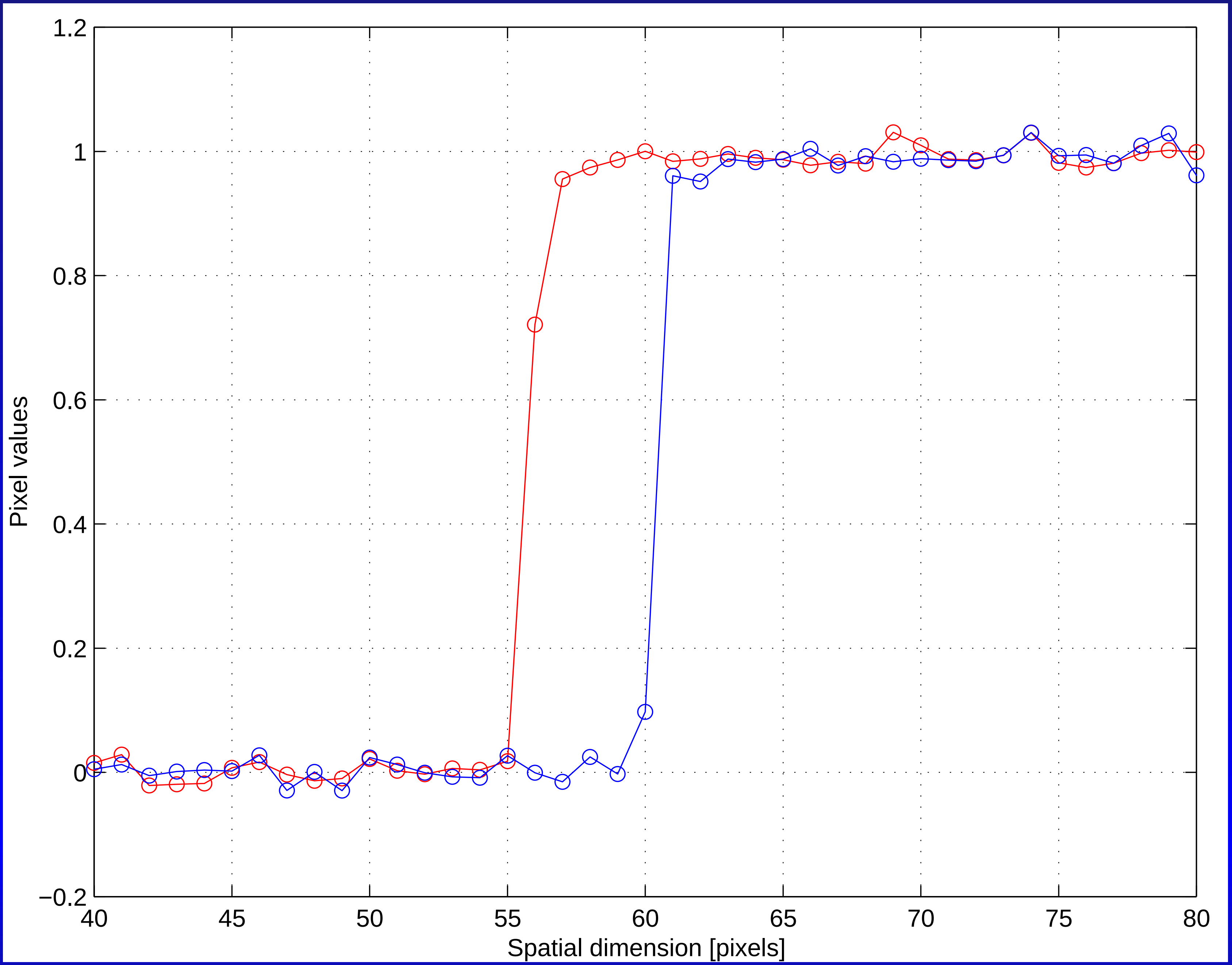
  - Δ – sampling rate for discrete-space approximation
  - $X'_m \approx X_m$ (Δ-quantized approximation to X_m)
  - Translation parameter X'_m – to be estimated

4. Simulated Imaging Process

- Three steps

  1. Low pass filtering (modeling optical system)
  2. Spatial integration and sampling (CCD sensor)
  3. Addition of Gaussian noise (CCD readout noise)

- $f[k, t_1] \longrightarrow \hat{f}[m, t_1]$
- $f[k, t_2] \longrightarrow \hat{f}[m, t_2]$
- In general, $\hat{f}[m, t_2] \neq$ translation of $\hat{f}[m, t_1]$ (texture)



1.2
1
0.8
0.6
0.4
0.2
0
−0.2
Pixel values
40
45
50
55
60
65
70
75
80
Spatial dimension [pixels]

5. Motion Estimation

- Standard block matching motion estimation

  $\Rightarrow$ 1-pixel *resolution*

- Interpolate image data $\hat{f}[m,t]$ by a factor of p

  $\Rightarrow \hat{f}_I[\frac{i}{p},t] = \hat{f}[m,t]$ for $i = mp$

- Standard block matching motion estimation on $\hat{f}_I[i,t]$

  $\Rightarrow \frac{1}{p}$-pixel *resolution* estimates

- Perfect: All errors $\pm\frac{1}{2p}$ pixels of true motion

- Resolution vs. Accuracy

- Find the highest motion estimation *accuracy* one can expect to achieve under realistic circumstances

- Choose largest p (highest resolution) such that motion estimates are accurate within some pre-specified bound

- Defn: Accurate to $\frac{1}{p}$ pixels $\Leftrightarrow$ α-% of motion estimation errors fall within $\left(-\frac{1}{2p}, +\frac{1}{2p}\right)$

- Choose α according to application

- Similarity Measures

$$\text{SAD}(d) = \sum_{i \in \mathcal{B}} \left| \hat{f}_I[i, t_1] - \hat{f}_I[i+d, t_2] \right|$$

$$\text{MSE}(d) = \frac{1}{|\mathcal{B}|} \sum_{i \in \mathcal{B}} \left\{ \hat{f}_I[i, t_1] - \hat{f}_I[i+d, t_2] \right\}^2$$

$$\text{NCF}(d) = \frac{\sum_{i \in \mathcal{B}} \hat{f}_I[i, t_1] \, \hat{f}_I[i+d, t_2]}{\left[\sum_{i \in \mathcal{B}} \hat{f}_I^2[i, t_1] \right]^{\frac{1}{2}} \left[\sum_{i \in \mathcal{B}} \hat{f}_I^2[i+d, t_2] \right]^{\frac{1}{2}}}$$

Motion Estimator Test Procedure

1. Choose *a-priori* motion X_m uniformly on $(-wX_s, wX_s)$

   w denotes search window size in pixels

2. Choose X_δ uniformly on $\left(-\frac{X_s}{2}, +\frac{X_s}{2}\right)$

3. Generate the scene data

$$f[k,t_1] = s(k\Delta + X_\delta) + g(k\Delta + X_\delta)$$

$$f[k,t_2] = s(k\Delta - X_m + X_\delta) + g(k\Delta - X_m + X_\delta)$$

4. Simulate the imaging process $\Rightarrow \hat{f}[m,t_1]$ and $\hat{f}[m,t_2]$

5. Interpolate $\Rightarrow \hat{f}_I[i,t_1]$ and $\hat{f}_I[i,t_2]$

6. Estimate $\hat{X}_m$ from interpolated data

7. Determine motion estimation error $\epsilon = \hat{X}_m - X_m$

5000 Trials for $1 \leq p \leq 20$ and each similarity measure

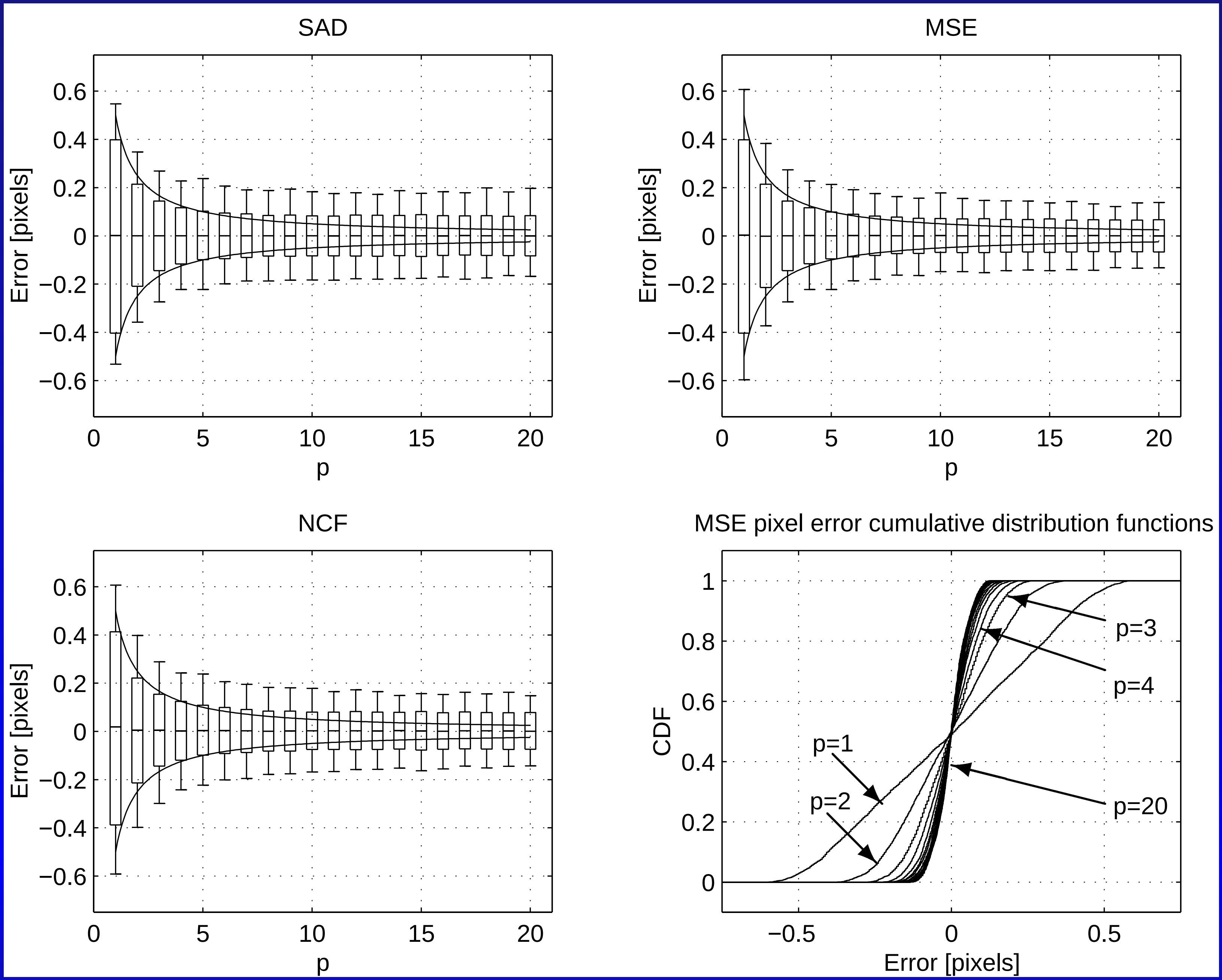
6. Results

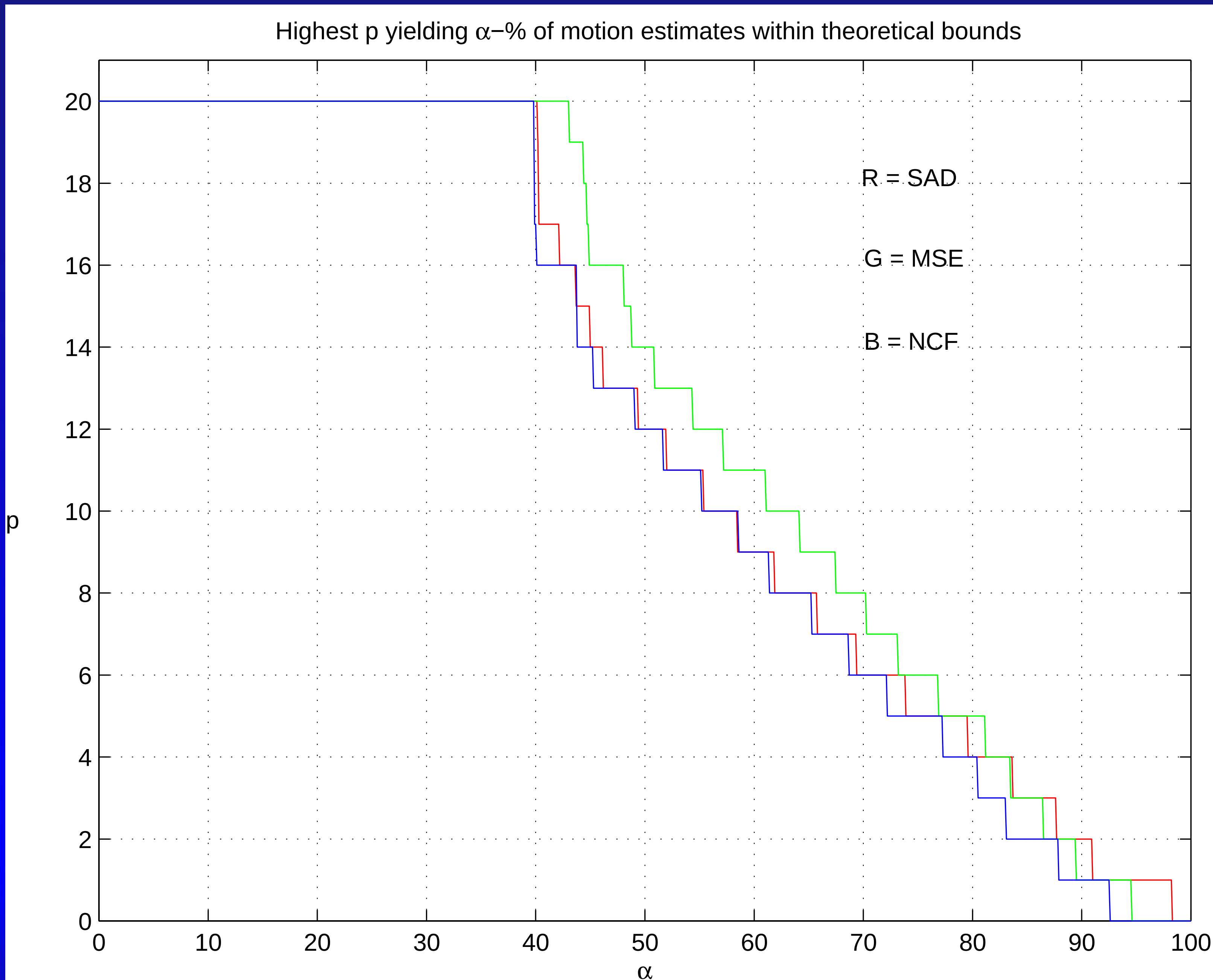
1. Fix α

- Compare actual performance with theoretical bounds
- $\Rightarrow$ diminishing returns for increasing p

2. Vary α

- Find highest p yielding α-% of errors within $\pm\frac{1}{2p}$ pixels of true motion
- $\Rightarrow$ accuracy / resolution trade-off


SAD
MSE
NCF
MSE pixel error cumulative distribution functions
Error [pixels]
p
CDF
p=1
p=2
p=3
p=4
p=20
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
20
1
0.8
−0.5
0.5



Highest p yielding α−% of motion estimates within theoretical bounds
R = SAD
G = MSE
B = NCF
p
α
0
2
4
6
8
10
12
14
16
18
20
0
10
20
30
40
50
60
70
80
90
100

7. Conclusions

- Seek best case performance of block-matching sub-pixel motion estimators

- Perfect: $\frac{1}{p}$-pixel motion estimator errors bounded within $\pm\frac{1}{2p}$ pixels of true motion

- Reality: Error approximates $\pm\frac{1}{2p}$ bounds for small p and *levels off* thereafter

- Resolution / Accuracy trade-off

  Level of acceptable error determines highest resolution estimator which may be used

Optical System

$$H_o(u,v) = \begin{cases} \frac{2}{\pi}\left\{\cos^{-1}\left(\frac{\rho}{\rho_c}\right) - \frac{\rho}{\rho_c}\left[1-\left(\frac{\rho}{\rho_c}\right)^2\right]^{\frac{1}{2}}\right\}, & \rho < \rho_c \\ 0, & \text{else} \end{cases}$$

$\rho = (u^2 + v^2)^{1/2}$ and u, v spatial frequency variables.

The radial cut-off frequency is given by,

$$\rho_c = \frac{1}{\lambda \cdot f/\text{number}}$$

We choose the wavelength in the visible spectrum as $\lambda = 670$nm, resulting in a cut-off at approximately 533 lines per mm.

CCD Focal Plane Array

$$h_{CCD}(x,y) = \frac{1}{X_s^2}\,\text{rect}\left(\frac{x}{X_s}\right)\text{rect}\left(\frac{y}{X_s}\right)$$

$$= \begin{cases} 1/X_s^2, & |x| \leq X_s/2 \text{ and } |y| \leq X_s/2 \\ 0, & \text{otherwise} \end{cases}$$

The two-dimensional Fourier transform is given by,

$$H_{CCD}(u,v) = \text{sinc}(X_s u)\,\text{sinc}(X_s v)$$

$$= \frac{\sin(\pi X_s u)}{\pi X_s u} \cdot \frac{\sin(\pi X_s v)}{\pi X_s v}$$

Sampling Considerations

- Simulated Imaging Sytem

  Optical System LPF $f_c = 533$ cycles/mm

  CCD spatial integration LPF $f_c = 111$ cycles/mm

  CCD sampling rate $f_c = 111$ samples/mm

  $\Rightarrow$ Undersampled

  Always the case where CCD aperture is anti-alias filter